

Electrodynamics in rotating and other accelerated frames of reference

B Chakraborty

Plasma Physics Group, Mathematics Department, Jadavpur University,
Calcutta-700 032, India

Abstract : A systematic description of the various aspects of electrodynamics in accelerated frames, with a special emphasis to that in frames rotating about a fixed axis, at a constant rate, is attempted here. The related basic problems and controversies are of such nature that these can not be covered briefly in books on classical electrodynamics. For this reason all authors of such books avoid any presentation of this electrodynamics, even for very small rotational velocities. These problems have been clearly exposed here.

Keywords : Electrodynamics in accelerated frames, constitutive equations, rotating frame

PACS Nos. : 02.40.-k, 03.50.-z, 04.40.+c

1. Introduction

This paper tries to provide a brief, but systematic description of the important aspects of electrodynamics in accelerated frames, with a special emphasis of that in rotating frames, and expose clearly the related basic problems which are still existing.

One such intriguing problem is that the covariant formulation of the constitutive relations (also called the state relations) is not unique. The possible formulations give correct relations between the field vectors in inertial systems only. However, this formulation does not ensure that these equations also hold in a non-inertial system. Several possible methods of writing down the covariant equations in noninertial frames exist. Choice of one of these for adoption should depend on physical justification only, which is still not available for guidance.

Failure to explain observations of rotating and stationary observers, relative to rotating conductors and other difficulties, has led to the view that the traditional renderings of the basic laws of electrodynamics are incomplete, and should be rectified for application to accelerated frames. For determination of the universally valid forms of these laws either the Maxwell equations are to be accepted as valid for accelerated frames and the constitutive relations are to be changed, or the Maxwell equations are to be modified without changing the constitutive relations, or both are to be modified for that purpose. The current tendency is that

the constitutive relations are to be changed and no change is envisaged for the Maxwell equations.

The elementary definition of physical quantities generally refers only to rectangular components, and must be supplemented before deciding on whether the physical vector is contravariant or covariant. The 4-force can be derived both as a contravariant 4-vector and as a covariant 4-vector. This is an example of the nonuniqueness of obtaining the derived fields from the prototypes or the domain quantities. However, the sum of inertial and gravitational forces may be tensorial. The Lorentz force is an example of a tensorial force field if it is regarded as a contravariant vector density.

To determine the constitutive relations that connect the EM field vectors in a medium, some general methods have been suggested by Tamm [1], Heer [2] and Khromykh [3], Post [4], Post and Yildiz [5] and Yildiz and Tang [6]. Volkov and Kiselev [7] further developed the method of Tamm [1]. All these covariant formulations of the state relations give correct state relations between the field vectors in inertial systems. But this identification does not ensure that these equations hold good as well in noninertial systems. The final choice, from among the several possible covariant formulations, should be decided by experimental indications which are not yet available.

2. Electrodynamical field equations in inertial frame

(a) *Vector form differential equations :*

	$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$	The homogeneous pair
		or
	$\nabla \cdot B = 0$	the source free pair
	$\nabla \times H = \frac{1}{c} \frac{\partial D}{\partial t} + \frac{4\pi}{c} j,$	The inhomogeneous pair
The Maxwell equations		or
	$\nabla \cdot D = 4\pi\rho$	the source full pair
		or
	$\nabla \cdot j + \frac{\partial \rho}{\partial t} = 0$	The continuity condition
	$F = \rho E + \frac{1}{c} (j \times B)$	The Lorentz force density
	$\left. \begin{aligned} D &= \epsilon E, \\ B &= \mu H \end{aligned} \right\}$	The constitutive relations for stationary isotropic media
	$j = \sigma E$	Ohm's law for stationary media

Gauge relations : $B = \nabla \times A, E = -\frac{1}{c} \frac{\partial A}{\partial t} +$
 $\nabla \cdot A = \frac{\epsilon\mu}{c} \frac{\partial \phi}{\partial t}$

Ohm's law in moving media : $J = \sigma \left(E + \frac{1}{c} (u \times B) \right)$

Equation of motion of a charge q : $\frac{dp}{dt} = q \left(E + \frac{1}{c} (u \times B) \right)$

(b) *Tensor form differential equations :*

The homogeneous pair of Maxwell equations becomes

$$\frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} = 0, \quad (\lambda, \mu, \nu) = (1, 2, 3, 4)$$

where $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$

$A_\mu \equiv (A, \phi), \quad \text{or} \quad A^\mu \equiv (A, -\phi)$
 $dx^\mu \equiv (dr, cdt), \quad \text{or} \quad dx_\mu \equiv (dr, -cdt)$

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{bmatrix}$$

The inhomogeneous pair of Maxwell equations becomes

$$\frac{\partial H^{\mu\nu}}{\partial x^\nu} = \frac{4\pi}{c} j^\mu, \quad (\mu, \nu) = (1, 2, 3, 4),$$

$$H^{\mu\nu} = \begin{bmatrix} 0 & H_z & -H_y & -D_x \\ -H_z & 0 & H_x & -D_y \\ H_y & -H_x & 0 & -D_z \\ D_x & D_y & D_z & 0 \end{bmatrix}$$

$$J^\mu \equiv (j, c\rho) \quad \text{or} \quad j_\mu \equiv (j, -c\rho)$$

The fundamental unit tensor is

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

so that the space-time metric is given by

$$\begin{aligned} ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dT^2 \\ &= dx^2 + dy^2 + dz^2 - (cdT)^2, \end{aligned}$$

dT is the proper time subinterval.

Ohm's law :

(a) In a stationary medium

$$j^\mu = \frac{\sigma}{c} F^{\mu\nu} U_\nu$$

where $U_\nu = (0, 0, 0, C)$

(b) In moving media

$$j^\mu + \frac{1}{c^2} (j^\nu U_\nu) U^\mu = \frac{\sigma}{c} F^{\mu\nu} U_\nu$$

where $U^\mu = \frac{dx^\mu}{dT}$ U^μ is the four velocity.

Equation of motion due to the Lorentz force :

$$\frac{dp^\mu}{dt} = \frac{q}{c} F^{\mu\nu} U_\nu$$

where $U^\mu = \frac{dx^\mu}{dt}$

$$\text{or} \quad \frac{dP^\mu}{dt} - \frac{q}{c} \frac{\partial A^\mu}{\partial x_\mu} = u^\nu, \quad P^\mu = p^\mu + \frac{q}{c} A^\mu$$

For the purpose of generalization to rotating frames a suitable form of this equation is

$$mc \frac{dw^\mu}{d\tau} = -\frac{q}{c} \eta_{\nu\xi} F^{\mu\xi} w^\nu$$

where $w^\mu = \frac{dx^\mu}{\sqrt{-ds^2}}$, m is the rest mass.

In vacuum, in presence of free charges, the constitutive relations are

$$\left. \begin{aligned} D &= E \\ B &= H \end{aligned} \right\} \text{Gaussian field identification}$$

The tensor form of the inhomogeneous pair of the Maxwell equation then take the form

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \frac{4\pi}{c} j^\mu$$

This Gaussian field identification is not valid in rotating and other accelerated frames because it will be evident from a subsequent discussion that even vacuum becomes anisotropic in these frames of reference.

Schiff [8] observed that if a spherical conductor is in rotation about a diameter, a magnetic field is seen to exist outside, and in the absence of rotation no field is observed outside the conductor. Moreover, detection, by experiments, of surface charge in the plates of rotating cylindrical condensers and emf of these plates have been reported as effects of rotation. These and some other related experimental findings could not be explained by the familiar theory of classical electrodynamics when naively extended to rotating frames.

Failure to explain the observations of rotating and stationary observers, relative to rotating conductors, has led to the question : Are the laws of electrodynamics in rotating and other accelerated systems same as, or different from these in inertial frames ? If the laws are different, then the question is : What is the correct form of these laws in rotating frames ?

The EM laws of induction and their application depend entirely on inertial frame observations involving the domain quantities. So, a primary role should be given to the integrals as the observable quantities, and not to the field quantities like E , H , D etc, which are point functions of space time. Two important divisions of electrodynamics therefore can be made in the following manner :

Universally valid Maxwell equations, which do not contain any material constant, but the number of which is not sufficient for solving problems.

Electrodynamics

Constitutive relations (or the state relations) which are complementary to the Maxwell equations, and which depend on the material constitution.

The field quantities are also divided into two parts in the following manner :

Observable quantities, also called primary quantities
or the domain quantities, example :

$$\begin{aligned} \text{emf} &= \int_{\Gamma} (E \cdot d\mathbf{l}) \\ \text{field flux} &= \iint_{\Sigma} (B \cdot d\mathbf{\Sigma}) \\ \text{current flux} &= I = \iint_{\Sigma} (j \cdot d\mathbf{\Sigma}) \end{aligned}$$

Two classes of
field quantities

Derived quantities or the locally valid point functions ;
example : E , H , D , B , J . These follow from the domain quantities of flux, obtain their properties as well from the flux quantities, and are functions of space and time.

The EM laws and their applications depend entirely on the inertial frame observations involving the domain quantities. A primary role should, therefore, be given to the integrals of fields for development of electrodynamics in rotating and other noninertial frames.

Distinction between the laws for the displacements and the consequent laws for the EM fields in inertial frames of reference. The Lorentz transformations for displacement are

$$dr_{\perp} = dr_{\perp}^{\circ},$$

$$dr_{\parallel} = \gamma(dr_{\parallel}^{\circ} - \beta c dt^{\circ}), \quad c dt = \gamma(c dt^{\circ} - \beta dr_{\parallel}^{\circ})$$

where the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = u/c$$

The consequent laws for EM fields :

$$H_{\perp} = \gamma \left(H_{\perp}^{\circ} - (\beta \times D_{\perp}^{\circ}) \right), \quad H_{\parallel} = H_{\parallel}^{\circ},$$

This distinction between space displacements and fields follows Faraday's law of EM induction : When an electrically conducting wire is moved in one direction the magnetic field perpendicular to that is affected, and the parallel magnetic field remains unchanged. This correspondence gives rise to the following relevant question : Is there any correspondence, full of promise, between velocity displacements (which are proportional to acceleration) and fields in accelerated frames ?

Analogy between EM fields and rotation parameters :

$$E = -\frac{m}{q}(\Omega \times (\Omega \times r)) - \frac{m}{q}(\dot{\Omega} \times r),$$

$$B = \frac{2m}{q}c\Omega,$$

$$\phi = \frac{m}{2q}(\Omega \times r)^2,$$

$$\frac{1}{c} \frac{\partial A}{\partial t} = -\frac{m}{q}(\dot{\Omega} \times r),$$

$$\rho = \frac{m\Omega^2}{2\pi q^2},$$

$$j = \frac{m}{4\pi q} \frac{\partial}{\partial t} [(\Omega \times (\Omega \times r)) + (\Omega \times r)]$$

This ρ and j satisfy the continuity condition

$$\nabla \cdot j + \frac{\partial \rho}{\partial t} = 0$$

Faraday's law of induction for rotation :

The rate of work done by a particle of unit mass in moving along a closed circuit, under the influence of forces of rotation, equals the rate of loss of flux of 2Ω through the surface enclosed by the circuit, where Ω is the angular velocity of rotation.

Relations of transformation of fields based on the Minkowski theory for moving media, correct upto the first order of the relative velocity u :

$$\mathbf{j} = \mathbf{j}^\circ - \rho^\circ \mathbf{u},$$

$$\rho = \rho^\circ,$$

$$\mathbf{A} = \mathbf{A}^\circ,$$

$$\phi = \phi^\circ + (\boldsymbol{\beta} \cdot \mathbf{A}^\circ),$$

$$\mathbf{E} = \mathbf{E}^\circ,$$

$$\mathbf{B} = \mathbf{B}^\circ,$$

$$\mathbf{H} = \mathbf{H}^\circ,$$

$$\mathbf{D} = \mathbf{D}^\circ + \frac{\epsilon}{c}(\mathbf{u} \times \mathbf{B}^\circ)$$

$$\mathbf{F} \text{ (Lorentz force)} = \rho \mathbf{E} + \frac{1}{c}(\mathbf{j} \times \mathbf{B}) = \rho^\circ \mathbf{E}^\circ + \frac{1}{c}(\mathbf{j}^\circ \times \mathbf{B}^\circ)$$

One consequence for $\boldsymbol{\vartheta} = (\Omega \times \mathbf{r})$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= (\nabla \cdot \mathbf{E}^\circ) + \nabla \cdot \left(\frac{1}{c}(\boldsymbol{\vartheta} \times \mathbf{B}^\circ) \right) \\ &= \frac{4\pi}{\epsilon} \rho + \frac{1}{c} \mathbf{B}^\circ \cdot (\nabla \times \boldsymbol{\vartheta}) - \frac{1}{c}(\boldsymbol{\vartheta} \cdot (\nabla \times \mathbf{B}^\circ)) \\ &= \frac{4\pi}{\epsilon} \rho + 4\pi \rho_s - \frac{1}{c}(\boldsymbol{\vartheta} \cdot (\nabla \times \mathbf{B}^\circ)) \end{aligned}$$

where $4\pi \rho_s = \frac{2}{c}(\Omega \cdot \mathbf{B}^\circ)$ when \mathbf{B}° is constant. ρ_s is called the spurious Schiff charge of rotation, per unit volume.

Galilean-Newtonian rules of transformation for translation at uniform rates :

$$\mathbf{r}^\circ = \mathbf{r} + \mathbf{u}t, \quad t^\circ = t$$

Galilean-Newtonian rules of transformation for rotation at uniform angular rate Ω :

$$x = x^0 \cos \Omega t^0 + y^0 \sin \Omega t^0,$$

$$y = -x^0 \sin \Omega t^0 + y^0 \cos \Omega t^0, \quad Z = Z^0, \quad t = t^0$$

$$\mathbf{r} = \mathbf{r}^0, \quad \theta = \theta^0 - \Omega t^0, \quad z = z^0, \quad t = t^0,$$

$$\boldsymbol{\vartheta} = (\Omega \times \mathbf{r}), \quad \boldsymbol{\beta} = \boldsymbol{\vartheta}/c, \quad r\Omega \ll c$$

$$\nabla^0 = \nabla, \quad (\nabla \cdot \boldsymbol{\vartheta}) = 0, \quad (\nabla \times \boldsymbol{\vartheta}) = 2\Omega, \quad (\mathbf{A} \cdot \nabla)\boldsymbol{\vartheta} = (\Omega \times \mathbf{A}),$$

$$\nabla(\vartheta \cdot A) = (\vartheta \cdot \nabla)A - (\Omega \times A) + (\vartheta \times (\nabla \times A)),$$

$$\left(\frac{dA}{dt} \right)_{K^0} = \left(\frac{dA}{dt} \right)_K + (\Omega \times A),$$

$$\frac{\partial A}{\partial t} \Big|_{\tilde{K}} = \frac{\partial A}{\partial t} - (\vartheta \cdot \nabla)A + (A \cdot \nabla)\vartheta \\ + \frac{\partial A}{\partial t} \cdot \nabla \times (\vartheta \times A) - \vartheta(\nabla \cdot A)$$

where K^0 is the nonrotating frame and K is the frame rotating at the constant angular rate Ω with respect to K^0 . These relations are valid for a fixed point P in the rotating frame K , so

$$\left(\frac{dA}{dt} \right)_K = \frac{\partial A}{\partial t};$$

and since P moves in the laboratory rest frame K^0 with velocity V , we can write

$$\left(\frac{dA}{dt} \right)_{K^0} = \frac{\partial A}{\partial t^0} + (\vartheta \cdot \nabla^0)A$$

Equation of motion of a single charged particle :

$$m \frac{du}{dt} = qE + \frac{q}{c}(u \times B) - 2m(\Omega \times u) \\ + m\{(\Omega \times r) \times \Omega\} - m(\dot{\Omega} \times r)$$

For Newtonian rotation

$$ds^2 = (dr)^2 + (rd\theta)^2 + (dz)^2 + 2\beta \cdot rd\theta \cdot cdt - (1 - \beta^2)(cdt)^2 \\ = (dr)^2 + \gamma^2(rd\theta)^2 + (dz)^2 - (cdt^*)^2 \\ = \gamma_{ij}dx^i dx^j - (cdt^*)^2$$

with $cdt^* = (cdt - \beta\gamma^2 rd\theta)/\gamma$,

$$\gamma_{ij} = \begin{matrix} 1 & 0 & 0 \\ 0 & \gamma^2 r^2 & 0 \\ 0 & 0 & 1 \end{matrix} \quad r = \left(1 - \frac{r^2 \Omega^2}{C^2}\right)^{-1/2}$$

$$\det \gamma_{ij} = \hat{y} = r^2 \gamma^2 = \frac{r^-}{1 - (r\Omega/c)^2}$$

We can write

$$ds^2 = \left[1 - \left\{ \frac{U}{c} + \frac{1}{c}(\Omega \times r) \right\}^2 \right] (cdt)^2 \\ = \left[1 - \left| \frac{u^0}{c} \right|^2 \right] (cdt)^2$$

where $u^0 = u + (\Omega \times r)$

Hence we find that

$$ds^2 = dl^2 - (cdt)^2$$

with $dl^2 = (dr)^2 + r^2(d\theta)^2 + (dz)^2$

Riemannian geometry of space of noninertial frames :

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = -(cdT)^2 \\ &= \gamma_{ij} dx^i dx^j - \left(a_i dx^i - \sqrt{-g_{44}} dx^4 \right)^2, \end{aligned}$$

$$\hat{\gamma} = \det \gamma_{ij} = \begin{vmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{vmatrix}$$

$$\det g_{\mu\nu} = g, \quad (\lambda, \mu, \nu) = (1, 2, 3, 4), \quad (i, j) = (1, 2, 3),$$

$$\gamma_{ij} = g_{ij} - \frac{g_{4i} g_{4j}}{g_{44}} = g_{ij} + a_i a_j$$

$$a_i = \frac{g_{4i}}{\sqrt{-g_{44}}}$$

Christoffel symbol :

$$\Gamma_{\mu\nu}^\eta = \frac{1}{2} g_{\lambda\eta} \left(\frac{\partial g_{\lambda\mu}}{\partial x^\nu} + \frac{\partial g_{\lambda\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda} \right)$$

Covariant formulation of electrodynamics of noninertial frames :

Field equations

$$\begin{aligned} \frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} &= 0 \\ \frac{\partial}{\partial x^\nu} (\sqrt{-g} I^{\mu\nu}) &= \frac{4\pi}{c} \sqrt{-g} j^\mu \end{aligned}$$

Equation of motion of a charge q

$$mc \left[\frac{dw^\mu}{\sqrt{-ds^2}} + \Gamma_{\nu\eta}^\mu w^\nu w^\eta \right] = \frac{q}{c} g_{\nu\xi} F^{\mu\xi} w^\nu$$

Continuity condition

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} j^\mu) = 0$$

Constitutive relations

(1) Shiozawa [9]

$$H^{\mu\nu}U_\nu = \varepsilon F^{\mu\nu}U_\nu, \quad {}^*F^{\lambda\mu}U_\mu = \mu {}^*H^{\lambda\mu}U_\mu,$$

where ${}^*F^{\alpha\beta} = \frac{1}{2} e^{\alpha\beta\gamma\delta} F_{\gamma\delta},$

$e^{\alpha\beta\gamma\delta}$ is the contravariant fourth order Levi Civita symbol,

$${}^*F^{\alpha\beta} = \begin{bmatrix} 0 & E_x & -E_y & B_x \\ -E_x & 0 & E_x & B_y \\ E_y & -E_x & 0 & B_z \\ -B_x & -B_y & -B_z & 0 \end{bmatrix}$$

${}^*F^{\alpha\beta}$ is the dual of $F^{\alpha\beta}$.

(2) Tamm [1] and Mandel'shtam [10]

$$H^{\mu\nu} = g^{\mu\alpha} g^{\gamma\beta} \xi_\alpha^\gamma \xi_\beta^\delta F_{\gamma\delta}$$

or

$$g_{\alpha\lambda} g_{\beta\gamma} H^{\alpha\beta} = \xi_\lambda^\gamma \xi_\gamma^\delta F_{\gamma\delta}$$

where the nonzero components of ξ_μ^λ are

$$\xi_1^1 = \xi_2^2 = \xi_3^3 = \frac{1}{\sqrt{\mu}}, \quad \xi_4^4 = \varepsilon\sqrt{\mu}$$

(3) Heer [2] and Khromykh [3]

$$H_{i4} = \varepsilon F_{i4},$$

$$F^{ij} = \mu H^{ij}$$

For moving media these constitutive relations generalize to the forms

$$g_{\alpha\xi} g_{\beta\eta} H^{\xi\eta} U^\beta = \varepsilon F_{\alpha\delta} U^\delta,$$

$$e_{\alpha\beta\gamma\delta} g^{\beta\xi} g^{\gamma\eta} F_{\xi\eta} U^\delta = \mu e_{\alpha\beta\gamma\delta} H^{\beta\gamma} U^\delta$$

where $e_{\alpha\beta\gamma\delta}$ is the covariant fourth order Levi Civita symbol.

(4) Post [4], Yildiz and Tang [6] (PYT)

$$H^{\alpha\beta} = \frac{1}{2} \chi^{\alpha\beta\gamma\delta} F_{\gamma\delta}$$

where $\chi^{\alpha\beta\gamma\delta}$ is the very general constitutive tensor of 4th order, which is specified in an inertial frame, for linear isotropic media, by the conditions

$$\gamma^{4^0} i^{4^0} j^{4^0} = -\varepsilon,$$

$$\chi^{i^0 j^0 i^0 j^0} = \frac{1}{\mu}$$

More specifically we can write

$$\chi^{4^0 1^0 4^0 1^0} = \chi^{4^0 2^0 4^0 2^0} = \chi^{4^0 3^0 4^0 3^0} = -\varepsilon,$$

$$\chi^{2^0 3^0 2^0 3^0} = \chi^{3^0 1^0 3^0 1^0} = \chi^{1^0 2^0 1^0 2^0} = \frac{1}{\mu}$$

For very small rotational velocities, the Galilean Newtonian rules of transformation to the rotating frame give

$$\text{in case (1)} \quad D = \varepsilon \{E - (\beta \times B)\},$$

$$B = \mu \{H + (\beta \times D)\};$$

$$\text{in case (2)} \quad D = \varepsilon E - \frac{1}{\mu} (\beta \times B),$$

$$B = \mu \{H + (\beta \times D)\};$$

$$\text{in case (3)} \quad D = \varepsilon E - (\beta \times H),$$

$$B = \mu H + (\beta \times E);$$

$$\text{in case (4)} \quad D = \varepsilon \{E - (\beta \times D)\},$$

$$B = \mu \{H + (\beta \times D)\}$$

For charges in vacuum, we put $\varepsilon = 1$, $\mu = 1$; then the constitutive relations in cases (1), (2) and (4), are $D = E - (\beta \times B)$, $B = H + (\beta \times D)$; and in case (3) these are $D = E - (\beta \times H)$, $B = H + (\beta \times E)$. So, even vacuum becomes anisotropic due to rotation.

Vector form differential equations of Maxwell in rotating frames :

$$\text{curl } E = -\frac{1}{c\sqrt{\gamma}} \frac{\partial}{\partial t} (\sqrt{\gamma} B)$$

$$\text{div } B = \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^i} (\sqrt{\gamma} B^i) = 0, \quad (i = 1, 2, 3)$$

$$\text{curl } H = \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial t} (\sqrt{\gamma} D) + \frac{4\pi}{c} \rho u$$

$$\text{div } D = \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^i} (\sqrt{\gamma} D^i) = 4\pi\rho, \quad (i = 1, 2, 3)$$

Vector form relations of transformation, correct upto the first order of rotation velocity, from tensor formulations of transformation laws [9] :

$$\nabla = \nabla', \quad t = t',$$

$$\rho = \rho',$$